Lecture 4

Learning Objectives

- To define a proposition
- To form a compound proposition using connectives
- To determine the truth values of compound propositions based on the truth values of their constituent propositions
- To use the different ways of expressing implication
- To determine the equivalence of two propositions

4.1 Logic

- Logic is a study of reasoning.

- Example 4.1.1:
  - All Taylor's students are hardworking.
  - Hardworking students will eventually smarter.
  - Therefore, all Taylor's students will eventually smarter.

- Logic is of no help to determine whether a statement is true or false.

- However, if the first two statements are true, logic assures that the third statement is true.

4.2 Propositions

- Logic involves statements, which are usually called propositions.
- A proposition is a statements can be either true or false but not both.
- Examples 4.2.1:
  Determine true or false for the following propositions:
  - Proposition P: All dogs are black.
  - Proposition Q: Obama is the president of Singapore.
  - Proposition R: Malaysia do not have winter.

Truth Values

- Every proposition has a truth value of either T (for "true") or F (for "false")
- If P is true, $\sim P$ is false.
- If P is false, $\sim P$ is true.

Connectives

- We combine propositions using connectives such as AND, OR, IMPLIES, IS EQUIVALENT TO (binary connectives)
- We also negate propositions using NOT (unary connective)
- Examples:
  - P and Q: All dogs are black, and Obama is the president of Singapore.
  - P or R: All dogs are black, or Malaysia do not have winter.

The Connective "OR"

- Exclusive-or (can't be both)
  - Today is Monday or Tuesday.
- Inclusive-or (Possible to be both)
  - You'll pass if you do very well in the mid-semester test or in the final exam.
    (You'll pass, if you do well in both exam.).
  - You'll know the answer if I tell you, or if someone else tell you.
    (You'll still know the answer, if we both tell you.).
- In Math's and in logic, "or" means "inclusive-or" unless otherwise specified.

Negation

- Negation of a proposition P, is the proposition NOT P.

- Example:
  - Let P: The moon is bigger than the sun.
  - The negation is – <u>not P</u>: The moon is NOT bigger than the sun.
    - Equivalently – The moon is smaller than (or possibly of equal size to) the sun.

- We use various symbols for "not":

$$\text{not } P$$
$$\sim P$$
$$\overline{\boldsymbol{P}}$$
$$\neg P$$

Example

Which of the following is the negation of "all camels have humps"?

1. No camels have humps.
2. Some camels have humps.
3. Some camels don't have humps.
4. All camels don't have humps.
5. Some camels don't know whether or not they have humps.

Implication

- If P is true then Q is true, or if P then Q.

$$P \Rightarrow Q$$

- P implies Q (Some texts use $P \rightarrow Q$)

- Examples: $x > 3 \Rightarrow x > 2$
  - If a number is bigger than 3 then it is bigger than 2.
  - If 3 is a negative number then the moon is made of green cheese.

    (This is true because 3 is not a negative number)

Implication

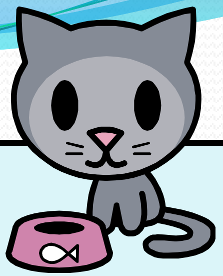
$$P \Rightarrow Q$$

If it is raining, then I am at home.

P	Q	$P \to Q$
True It’s raining	True I am at home	True
True It’s raining	False I am not at home	False
False It’s not raining	True I am at home	True
False It’s not raining	False I am not at home	True

P: It is a cat.

Q: It is an animal.



Ways to Write Implication $P \Rightarrow Q$	Example
P implies Q	It is a cat <u>implies</u> that it is an animal.
if P then Q	<u>If</u> it is a cat <u>then</u> it is an animal.
if P, Q	<u>If</u> it is a cat, it is an animal.
Q if P	It is an animal <u>if</u> it is a cat.
P only if Q	It is a cat <u>only if</u> it is an animal.
P is sufficient for Q	Being a cat <u>is sufficient for</u> being an animal.
Q is necessary for P	Being an animal <u>is necessary for</u> being a cat. (It can't be a cat if it is not an animal.)
Q (is true) whenever P (is true)	It is an animal <u>whenever</u> it is a cat.

More About Implications

Let $P \Rightarrow Q$ be an implication.

Its...	is...	Example
Converse	$Q \Rightarrow P$	If it is an animal, then it is a cat.
Contrapositive	$\sim Q \Rightarrow \sim P$	If it is not an animal, then it is not a cat.
Inverse	$\sim P \Rightarrow \sim Q$	If it is not a cat, then it is not an animal.

More About Implications

	P: Bruce is a rabbit. *Q*: Bruce is an animal.		
$P \Rightarrow Q$	If Bruce is a rabbit then Bruce is an animal.	[True]	
Converse $Q \Rightarrow P$	If Bruce is an animal then Bruce is a rabbit.	False	An implication may be true while its converse is false (and vice versa)
Contrapositive $\sim Q \Rightarrow \sim P$	If Bruce is not an animal then Bruce is not a rabbit.	True	It can be shown that every implication is logically equivalent to its contrapositive. $P \Rightarrow Q \equiv \sim Q \Rightarrow \sim P$

“Some”

- In formal logic, “some” means “at least one”.
- It includes the possibility of “all” as a special case.

- Example: Some dogs are animals.
  - True.
  - All dogs are animals. Therefore, some dogs are animals

Quantifiers

- These are words or phrases like:

  some all none there exists most

- They get applied to variables to create more complicated statements

Statement	**Variable**	**With Quantifiers**	**True/False**
Integers are positive.	Integers	Some integers are positive.	True
Integers have real square roots.	Integers	All integers have real square roots.	False
Rational numbers equal π.	Rational numbers	No rational numbers equal π.	True
Positive integers are smaller than every other positive integer.	Positive integers	There exists a positive integer which is smaller than every other positive integer.	True

Examples

Which of the following are true?

1. Some integers are real numbers.
2. Most integers are real numbers.
3. All integers are real numbers.
4. Some real numbers are irrational.
5. All real numbers are irrational.

Equivalence

- If whenever P is true, Q is true, and also whenever Q is true, P is true, We say that P is equivalent to Q

$$P \Leftrightarrow Q$$

$$P \leftrightarrow Q$$

- In words:

P is equivalent to Q
P if and only if Q
P is necessary and sufficient for Q

Note on Equivalence

$$P \Leftrightarrow Q \quad \text{means} \quad \begin{cases} P \Rightarrow Q \\ \text{and} \\ Q \Rightarrow P \end{cases}$$

P: He study in Taylor's
Q: He is a student of Taylor's

$$P \Rightarrow Q$$

If he study in Taylor's, then, he is a student of Taylor's

$$Q \Rightarrow P$$

If he is a student of Taylor's, then, he study in Taylor's

Therefore, P and Q are equivalence. $P \Leftrightarrow Q$